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EXPANSION OF THE LUMPED PARAMETER METHOD TO NONLINEAR, SOIL-STRUCTURE INTERACTING DYNAMIC SYSTEMS BY MEANS OF A MULTI-OBJECTIVE OPTIMIZATION ALGORITHM

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Abstract. The computational demands of the soil structure interaction analysis in scenarios such as the seismic risk assessment of a structure has led the civil engineering community to the development of a variety of methods towards the model order reduction of the soilstructure dynamic system. Different approaches have been proposed in the past as computationally viable alternatives to the FEM simulation of the complete structure-soil domain such as the nonlinear lumped spring, the macroelement method and the substructure partition method. Yet, with few exceptions, no approach was capable of capturing simultaneously the frequency-dependent dynamic properties along with the nonlinear behavior of the condensed segment of the overall soil - structure system, an essential step for the accurate assessment of a structure's performance under a seismic hazard. To this end, a dual frequency- and intensity-dependent expansion of the lumped parameter modeling method is proposed in the current paper, materialized through a multi-objective algorithm, capable of fully capturing the behavior of the nonlinear dynamic system of the condensed segment. The efficiency of the proposed approach is validated for the case of an existing bridge, wherein the seismic response is comparatively assessed for both the proposed method and the detailed finite element model.

1 INTRODUCTION

As has been observed in multiple occasions in the past [1], the supporting soil can play an essential role on the superstructure's behavior during the occurrence of a significant earthquake incident. The soil–structure interaction effect under such a dynamic hazard can lead to unexpected structural behavior and thus a realistic representation of the semi-infinite soil domain during the simulation of the soil structure interacting system is considered an essential prerequisite to an accurate analysis.

Numerous approaches have been established in the past capable of coping with the soil structure interaction simulation from a number of different perspectives. The truncated soil domain finite element method is an example of a direct approach on the simulation of the aforementioned effect where a detailed behavior of the soil – foundation system including material and geometrical complexity can be obtained. However such an approach can be computationally burdening for the analysis of complex structures, while in regard to analysis frameworks with prohibited computational cost such as the hazard risk assessment of a structure, the truncated FEM method is not a viable option.

In the light of the aforementioned limitations, the most common approach is to sacrifice the modeling refinement in terms of subsoil domain size by reducing the order of the system and primary focus on refined models tailored to capture damage at the structural and foundation components, as well as a result of geometric nonlinearities that arise from gap closure and stopper activation. Such a task can be accomplished through a partition approach of the overall dynamic system in segments, where the domain of the soil and foundation is significantly condensed on its internal degrees of freedom or completely replaced by a simplified representation, while the superstructure is left unaltered. Numerous implementations on the substructure modeling approach have been proposed in the past among which the macroelement method tends to stand out as an effective tool capable of coping with complex constitutive laws and geometrical nonlinearities.

The macroelement approach, a concept initially introduced by Montrasio et al [2] and further developed by a number of different research groups [3–5], has successfully provided with an accurate yet low in computational effort representation of the inelastic behavior of the soil-foundation domain. Although the macroelement approach emulates with great detail the different mechanisms triggered during a quasi-static excited simulation, it is common in the literature to either completely neglect or oversimplify the dynamic traits of the foundation soil domain through the use of complementary Kelvin –Voigt components. As such components are only limited to accuracy along a specific frequency their use can lead to significant error on the behavior of the structure under dynamic excitation as illustrated in [6].

A successful attempt on the incorporation of frequency dependency in macroelements has been recently accomplished by Chai et al [7]. However as the proposed method is mainly focused on the frequency dependent traits of the system in the elastic domain, oscillations in higher intensity regions, are represented by unexplored dynamic properties, thus suggesting potential misrepresentation of the soil foundation domain under specific circumstances. To this end a frequency dependent macroelement method with emulated dynamic properties among different levels of intensity is proposed in the current study. The viscoelastic dynamic properties of the soil foundation domain along a broad frequency spectrum has been successfully emulated in time domain simulations through the use of the lumped parameter modeling method [8–11] thus due to the method's past success in regard to perseverance of the original system's accuracy, stability and passivity, the LP method is selected as the basis of the newly proposed frequency dependent macroelement.

The objectives of the present study are therefore as follows: (a) the development of an extraction technique capable of selecting representative properties of the dynamic system along different intensity levels (b) the development of a complete procedure capable of capturing the aforementioned dynamic traits on the available intensity levels; and (c) a numerical verification of the proposed procedure through the use of a simplified single DOF shallow foundation study. The proposed procedure, as well as a set of illustrative applications, is described in the following paragraphs.

2 EXTRACTION OF SYSTEM DYNAMIC PROPERTIES AMONG DIFFERENT INTENSITY LEVELS

An essential step towards the expansion of the Lumped parameter modelling method to inelastic dynamic systems is the derivation of representative dynamic traits of the system in different levels of excitation intensity. In the following paragraphs a dynamic trait extraction approach is presented, implementing the linearization and dynamic condensation of the soil – foundation segment of the system on different representative segment state variable combinations.

2.1 System Linearization on a selected Variables State

The mathematical representation of the superstructure – foundation – semi-infinite soil domain system can be illustrated in the classic ode formulation of equation (1) after the appropriate geometrical discretization of the original PDE equation system.

$$\mathbf{M} \cdot \ddot{\mathbf{u}} + \mathbf{C} \dot{\mathbf{u}} + \mathbf{f} (\mathbf{u}) = \mathbf{F}$$
(1)

The variables **M** and **C** denote the mass and damping coefficient matrices of the system, $\mathbf{f}(\mathbf{u})$ denotes the nonlinear force to displacement relation vector while **F** is the external loading vector applied to the respective DOFs. It is possible to derive a linearized model of the aforementioned nonlinear dynamic system around a preselected state \mathbf{u}_{o} , through the Taylor expansion of the nonlinear returning force component $\mathbf{f}(\mathbf{u})$ as illustrated in equation (2). The linearized model accuracy is highly correlated with the distance of the dynamic system current variable state from the variable state **uo**.

$$\mathbf{M} \cdot \ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \dot{\mathbf{f}}(\mathbf{u}_{o}) \cdot (\mathbf{u} - \mathbf{u}_{o}) + \mathbf{f}(\mathbf{u}_{o}) = \mathbf{F}$$

$$(2)$$
Where $\mathbf{u} \in [\mathbf{u}_{o} - \varepsilon, \mathbf{u}_{o} + \varepsilon]$

The overall system can be partitioned into individual segments according to the previously presented substructure partition method, where the soil –foundation segment expected to be reduced is designated as the condensed segment. The nonlinear returning force equation terms of the condensed segment are replaced by their representative 1st order Taylor expansion corresponding to a specific variable state region $\mathbf{u}_o = [\mathbf{u}_o^i, \mathbf{u}_o^{soil}]$.

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}_{ss} \\ \mathbf{u}_{i} \\ \mathbf{u}_{soil} \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{ii}^{ss} + \mathbf{M}_{ii}^{soil} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{soil} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_{ss,ss} & \mathbf{C}_{ss,i} & \mathbf{0} \\ \mathbf{C}_{i,ss} & \mathbf{C}_{ii}^{ss} + \mathbf{C}_{ii}^{soil} & \mathbf{C}_{i,soil} \\ \mathbf{0} & \mathbf{C}_{soil,i} & \mathbf{C}_{soil,soil} \end{bmatrix}$$
$$\mathbf{f}(\mathbf{u}) = \begin{bmatrix} \mathbf{f}_{s}(\mathbf{u}_{ss}, \mathbf{u}_{i}) \\ \mathbf{f}_{i}^{s}(\mathbf{u}_{ss}, \mathbf{u}_{i}) \\ \mathbf{0} & \mathbf{f}_{i,i}(\mathbf{u}_{0}) & \mathbf{f}_{i,soil}(\mathbf{u}_{0}) \\ \mathbf{0} & \mathbf{f}_{soil,i}(\mathbf{u}_{0}) & \mathbf{f}_{soil,soil}(\mathbf{u}_{0}) \end{bmatrix} \cdot \begin{bmatrix} \mathbf{u}_{ss} \\ \mathbf{u}_{i} \\ \mathbf{u}_{ve} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{X}_{i} \\ \mathbf{X}_{soil} \end{bmatrix}$$
$$\mathbf{X}_{i} = \mathbf{f}_{i}^{soil}(\mathbf{u}_{0}) + \mathbf{\dot{f}}_{i,i}(\mathbf{u}_{0}) \cdot \mathbf{u}_{o}^{i} + \mathbf{\dot{f}}_{i,soil}(\mathbf{u}_{0}) \cdot \mathbf{u}_{o}^{soil}$$
$$\mathbf{X}_{soil} = \mathbf{f}_{soil}(\mathbf{u}_{0}) + \mathbf{\dot{f}}_{soil,i}(\mathbf{u}_{0}) \cdot \mathbf{u}_{o}^{i} + \mathbf{\dot{f}}_{soil,soil}(\mathbf{u}_{0}) \cdot \mathbf{u}_{o}^{soil}$$
(3)

The expansion of the system presented in equation (1) according to the selected segmentation and condensed segment linearization is illustrated in equation (3). The overall system's partitions are notated as ss, soil, and i corresponding to the superstructure segment, the soildomain - foundation segment and the interface DOFs, respectively. The matrix system of equation (3) is expanded into the equation formulas (4-6), where the equation terms corresponding to internal condensed segment DOFs are appropriately transformed in the frequency domain.

$$\mathbf{M}_{ss}\ddot{\mathbf{u}}_{ss} + \mathbf{C}_{ss,ss}\dot{\mathbf{u}}_{ss} + \mathbf{C}_{ss,i}\dot{\mathbf{u}}_{i} + \mathbf{f}_{s,ss}(\mathbf{u}_{ss},\mathbf{u}_{i}) = \mathbf{F}_{s}$$
(4)

$$\mathbf{M}_{ii}^{ss} \ddot{\mathbf{u}}_{i} + \mathbf{C}_{ii}^{ss} \dot{\mathbf{u}}_{i} + \mathbf{f}_{s,ii} (\mathbf{u}_{ss}, \mathbf{u}_{i}) + \mathbf{C}_{i,ss} \dot{\mathbf{u}}_{ss} + \mathcal{F}_{i,ss}^{soil} \ddot{\mathbf{u}}_{i} + \mathbf{C}_{ii}^{soil} \dot{\mathbf{u}}_{i} + \dot{\mathbf{f}}_{i,i} (\mathbf{u}_{o}) \cdot \mathbf{u}_{i} + \mathbf{C}_{i,soil} \dot{\mathbf{u}}_{soil} + \dot{\mathbf{f}}_{soil,i} (\mathbf{u}_{o}) \cdot \mathbf{u}_{soil} + \mathbf{X}_{i} (\mathbf{u}_{o})) = \mathbf{F}_{i}$$

$$(5)$$

$$\mathcal{F}(\mathbf{M}_{soil}\ddot{\mathbf{u}}_{soil} + \mathbf{C}_{soil,i}\dot{\mathbf{u}}_{i} + \mathbf{C}_{soil,soil}\dot{\mathbf{u}}_{soil} + \dot{\mathbf{f}}_{soil,i}(\mathbf{u}_{o}) \cdot \mathbf{u}_{ii} + \dot{\mathbf{f}}_{soil,soil}(\mathbf{u}_{o}) \cdot \mathbf{u}_{soil} + \mathbf{X}_{soil}(\mathbf{u}_{o})) = \mathbf{0}$$
(6)

The notations \mathscr{F} and \mathscr{F}^{-1} denote the Fourier and the inverse Fourier transformation. Through the appropriate transformations equation 5 and 6 can take the form of equations 7 and 8 respectively.

$$\mathbf{M}_{ii}^{ss} \ddot{\mathbf{u}}_{i} + \mathbf{C}_{ii}^{ss} \dot{\mathbf{u}}_{i} + \mathbf{C}_{i,ss} \dot{\mathbf{u}}_{ss} + \mathbf{f}_{s,tt} (\mathbf{u}_{ss}, \mathbf{u}_{i}) + \mathscr{F}^{-1} \left(\mathbf{S}_{ii}^{soil} \mathbf{U}_{i} + \mathbf{S}_{i,soil} \mathbf{U}_{soil} + \mathbf{X}_{i} (\mathbf{u}_{o}) \cdot 2\pi \cdot \delta(\omega) \right) = \mathbf{0} \quad (7)$$

$$\mathbf{S}_{soil,soil}\mathbf{U}_{soil} + \mathbf{S}_{soil,i}\mathbf{U}_{i} + \mathbf{X}_{soil}(\mathbf{u}_{o}) \cdot 2\pi \cdot \delta(\omega) = 0$$
(8)

Function $\delta(\omega)$ corresponds to the Dirac delta function, notation \mathbf{S}_{ij} represents the current impedance notation $-\mathbf{M}_{ij}\omega^2 + \mathbf{C}_{ij}\omega \mathbf{i} + \mathbf{\dot{f}}_{i,j}(\mathbf{u}_o)$ while $\mathbf{U} = \mathscr{F}(\mathbf{u}_j)$. The dynamic system of equations (3) can now be significantly reduced in size through the elimination of the Fourier transformed displacement vector \mathbf{U}_{soil} as illustrated in equation 9.

 $\mathbf{M} \cdot \ddot{\mathbf{u}} + \mathbf{C} \dot{\mathbf{u}} + \mathbf{f} (\mathbf{u}_{a}) + \mathbf{P} = \mathbf{F}$

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}_{ss} \\ \mathbf{u}_{i} \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{ii}^{ss} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_{ss,ss} & \mathbf{C}_{ss,i} \\ \mathbf{C}_{i,ss} & \mathbf{C}_{ii}^{ss} \end{bmatrix}$$
$$\mathbf{f}(\mathbf{u}) = \begin{bmatrix} \mathbf{f}_{s,ss}(\mathbf{u}_{ss},\mathbf{u}_{i}) \\ \mathbf{f}_{s,tt}(\mathbf{u}_{ss},\mathbf{u}_{i}) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathscr{F}^{-1}(\mathbf{S}_{ve,sg}) * \mathbf{u}_{i} \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} \mathbf{0} \\ \mathscr{F}^{-1}(\mathbf{RF} \cdot \sqrt{2\pi} \cdot \delta(\omega)) \end{bmatrix}$$
$$\mathbf{RF} = \mathbf{S}_{i,soil} \cdot \mathbf{S}_{soil,soil}^{-1} \cdot \mathbf{X}_{soil}(\mathbf{u}_{o}) + \mathbf{X}_{i}(\mathbf{u}_{o})$$
$$\mathbf{S}_{reduced} = \mathbf{S}_{i,i}^{soil}(\mathbf{u}_{o}) - \mathbf{S}_{i,soil}(\mathbf{u}_{o}) \cdot \mathbf{S}_{soil,soil}^{-1}(\mathbf{u}_{o}) \cdot \mathbf{S}_{soil,i}(\mathbf{u}_{o})$$
(9)

The extracted complex matrices $\mathbf{RF}(\mathbf{u}_0)$ and $\mathbf{S}_{reduced}(\mathbf{u}_0)$ are capable of accurately emulating the local dynamic behavior traits of the foundation-soil condensed segment in regard to small trajectories of the segment DOFs near the linearization state space \mathbf{u}_{o} .

2.1 Truncated selection of condensed system

Due to the nonfinite number of different state variable sets of the soil foundation segment, it is numerically impossible to retrieve the condensed complex matrix pairs **RF** and **S**_{reduced} for each possible combination. As a result a selection of representative state variable sets in connection to the behavior of the interface DOFs is essential.

An effective yet computationally non-burdening truncation on the possible state variable combinations can be accomplished through the use of the static pattern of deformation of the condensed segment corresponding to a specific configuration of the interface DOFs state-space \mathbf{u}_i . The current stiffness $\dot{\mathbf{f}}_{soil,soil}$ matrices and residual force \mathbf{f}_i^{soil} , \mathbf{f}_{soil} vectors corresponding to a specific interface DOF configuration are calculated from the condensed segment static equation system under the externally imposed displacement values of \mathbf{u}_i . The process is repeated for a discrete number of state variable sets \mathbf{u}_i within the expected domain of displacement for the interface DOFs.

3. INELASTIC LUMPED PARAMETER MODEL

Since the inelastic dynamic behavior of the condensed soil foundation segment can now be extracted in the form of a preselected number of complex matrix doublets **RF** and **S**_{reduced}, it is possible to develop a modified LP model assembly capable of emulating such behavior. The predefined Type 3 LP design proposed by Saitoh [12], is selected as the basis of the newly developed inelastic LP model. The proposed LP model for the scenario of a single interface DOF is illustrated in figure (1).



Figure 1. Physical representation of the proposed Lumped parameter Model

The LP model consists of one inelastic base component S_b capable of emulating the quasi-static properties of the condensed soil foundation segment, while additional complementary components are accordingly calibrated in order to match the intensity dependent dynamic traits of the system. The base component can be utilized by elastoplastic or hypoplastic macroelements proposed in the literature in accordance to the case specific foundation. In regard to the complementary components, conventional solutions are not capable of providing the sufficient accuracy during the LP model calibration process, as the assembly is now expected to emulate the dynamic behavior of the targeted system in different levels of intensity. To this end inelastic externally controlled components $s_{comp,i}$ are integrated in the LP model as presented in the following paragraphs, while the components $c_{i,up}$, $c_{i,down}$ and m_i follow the conventional definition of dashpot and mass components respectively.

3.1 Derivation of externally controlled inelastic components

The $s_{comp,i}$ components of the proposed LP assembly follow the multi-linear inelastic constitutive law illustrated in equation (10). The aforementioned components current stiffness is associated with variables representing the plastic state of the base component.

$$\dot{F}_{comp} = k_{comp}(\mathbf{a}_{ex}) \cdot \dot{u}$$
where $k_{comp}(\mathbf{a}_{ex}) = \begin{cases} k_o & \text{if } \mathbf{a}_{ex} < \mathbf{c}_o \text{ or } Unloading \\ \vdots \\ k_n & \text{if } \mathbf{c}_{n-1} < \mathbf{a}_{ex} < \mathbf{c}_n \end{cases}$
(10)

The variable Vector \mathbf{a}_{ex} corresponds to the base component S_b parameters at the current state of the system, \mathbf{c}_i are constant value vectors controlling the regions for which \mathbf{a}_{ex} corresponds to different spring stiffness k_i , while u is the change rate of the component's displacement.

The selection of an externally controlled component lies within the nature of the extracted dynamic traits. As presented in the previous sections, each extracted doublet of **RF** and $S_{reduced}$ has been affiliated with a specific combination of interface DOF displacements. The interface DOF controlled components within the LP model assembly possess an independent stiffness value corresponding to each interface displacement combination. As a result the calibration process can be significantly more efficient in comparison to the use of conventional multi-linear inelastic springs. The components are numerically implemented to the LP assembly through the explicit Runge-Kutta iterative method.

3.2 LP model Impedance Functions

Through the definition of the base and complementary components of the assembly it is now possible to generate the Matrix Expression of the proposed LP model in regard to a single DOF interface, as illustrated in equation (11).

$$\mathbf{f}_{LP}(\mathbf{u}) = \begin{bmatrix} f_{base}(\mathbf{u}) + f_{comp,1}(\mathbf{u}) \cdots + f_{comp,N}(\mathbf{u}) \\ f_{comp,1}(\mathbf{u}) \\ \vdots \\ f_{comp,N}(\mathbf{u}) \end{bmatrix} \quad \mathbf{M}_{LP} = \begin{bmatrix} M & 0 & \dots & 0 \\ 0 & m_1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & m_N \end{bmatrix}$$

$$\mathbf{C}_{LP} = \begin{bmatrix} C + c_{up,1} \dots + c_{up,N} & -c_{up,1} & \dots & -c_{up,N} \\ - c_{up,1} & c_{up,1} + c_{down,1} & \vdots \\ \vdots & & \ddots & 0 \\ - c_{up,N} & \dots & 0 & c_{up,N} + c_{down,N} \end{bmatrix}$$
(11)

The variable vector **u** corresponds to the displacement of the LP model's DOFs, \mathbf{M}_{LP} represents the mass matrix of the LP model, \mathbf{C}_{LP} represents the damping matrix of the LP model while \mathbf{f}_{LP} denotes the nonlinear force to displacement relation vector.

$$S_{LP}^{j} = K^{j} - M \cdot \omega^{2} + C \cdot \omega i + \sum_{i=1}^{N} \left(\frac{(k_{comp,i}^{j} + c_{up,i} \cdot \omega i)(-m_{i} \cdot \omega^{2} + c_{down,i} \cdot \omega i)}{k_{comp,i}^{j} + c_{up,i} \cdot \omega i - m_{i} \cdot \omega^{2} + c_{down,i} \cdot \omega i} \right)$$
(12)

$$RF_{LP}^{j} = X + \sum_{i=1}^{N} \left(\frac{(k_{comp,i}^{j} + c_{up,i} \cdot \omega i) \cdot x_{i}}{k_{comp,i}^{j} + c_{up,i} \cdot \omega i - m_{i} \cdot \omega^{2} + c_{down,i} \cdot \omega i} \right)$$

$$where: X = f_{base}(\mathbf{u}^{j}) + (K^{j}) \cdot u_{base}^{j} + \sum_{i=1}^{N} \left(f_{comp,i}(\mathbf{u}^{j}) - k_{comp,i}^{j} \cdot u_{i}^{j} \right)$$

$$x_{i} = f_{comp,i}(\mathbf{u}^{j}) - k_{comp,i}^{j} \cdot u_{base}^{j} + k_{comp,i}^{j} \cdot u_{i}^{j}$$
(13)

In a similar manner to the previously presented linearization process of the soil foundation segment, it is possible to derive a linearized model of the LP dynamic system around a prese-

lected state \mathbf{u}^{j} , through the Taylor expansion of the nonlinear returning force vector \mathbf{f}_{LP} . Through the affiliation of the interface DOF displacements to a variable state combination of the LP model it is possible to extract the model's \mathbf{S}_{LP} and \mathbf{RF}_{LP} matrix doublets, representing the dynamic behavior of the LP model in accordance to a specific interface DOF displacement combination. The impedance and returning force functions S_{LP} and RF_{LP} for the single interface DOF LP model are illustrated in equations (Eq. 12-13).

4. PROPOSED MODEL CALLIBRATION

4.1 Model order reduction through LP model calibration

The model order reduction approach selected for the current study is materialized through the derivation of a reduced model capable of approximating the behavior of the targeted complex system. It is possible to formulate the model order reduction approach in the optimization problem illustrated in equation (12).

$$\min_{S_r} f(M, M_r)$$
subject to $g(M, M_r) \le 0$
 $h(M, M_r) = 0$
(14)

The variable S represents the targeting dynamic system ODE formulation, S_r is the reduced order system possessing a lower number of state variables in comparison to S, function f specifies the behavioral similarities between the targeted and reduced system, while functions g and h are inequality and equality constraints of the optimization problem, imposing attractor behavior and maintaining specific structure properties of the system S.

The optimization problem of (eq. 14) can be reformulated on the specific scenario of the SSI system order reduction, through the appropriate derivations of the previously presented objective function and imposed constraint terms. In that case the soil foundation segment of the overall system is significantly condensed through the use of the inelastic LP model presented in the previous section as an adequate replacement. The order reduction problem takes the following multi objective optimization form as illustrated in equations (eq. 15-16).

$$\min_{\mathbf{x}} \left(f_1(x), \dots f_N(x) \right)$$

subject to $\mathbf{x} \ge 0$ (15)

$$f_{j}(\mathbf{x}) = \sum_{i=1}^{N} \left[\operatorname{Re}\left(S_{Tar}^{j}(\omega_{i})\right) - \operatorname{Re}\left(S_{LP}^{j}(\omega_{i},\mathbf{x})\right) \right]^{2} + \sum_{i=1}^{N} \left[\operatorname{Im}\left(S_{Tar}^{j}(\omega_{i})\right) - \operatorname{Im}\left(S_{LP}^{j}(\omega_{i},\mathbf{x})\right) \right]^{2} + \sum_{i=1}^{N} \left[\operatorname{Re}\left(RF_{Tar}^{j}(\omega_{i})\right) - \operatorname{Re}\left(RF_{LP}^{j}(\omega_{i},\mathbf{x})\right) \right]^{2} + \sum_{i=1}^{N} \left[\operatorname{Im}\left(RF_{Tar}^{j}(\omega_{i})\right) - \operatorname{Im}\left(RF_{LP}^{j}(\omega_{i},\mathbf{x})\right) \right]^{2} \right]^{2}$$
(16)

Index *j* denotes a specific interface DOF variable state, S^{j}_{tar} and S^{j}_{LP} indicates the impedance functions of the targeted and reduced LP system respectively on a given variables state *j*, while RF^{j}_{Tar} and RF^{j}_{LP} indicates the returning force complex functions of the targeted and reduced LP system respectively on a given variables state *j*.

4.2 Proposed algorithmic approach for the optimization process

The scalarization of the multiobjective optimization problem illustrated in equations (eq. 15-16) is selected as the most effective approach, as the problem's high number of objective functions can lead to computationally burdening solutions when a posteriori methods are introduced. The weighted-sum method initially proposed by Zadeh [13], is a traditional, popular method following an a priori preference on the selection of a Pareto optimal through appropriate importance related weighting of the objective functions. The initial optimization problem takes the following single objective form as illustrated in equation (eq. 17).

$$\min_{\mathbf{x}} g(\mathbf{x}) = \sum_{j=1}^{\infty} w_j \cdot f_j(\mathbf{x})$$
subject to $\mathbf{x} \ge 0$
(17)

As the objective function commonly behaves in a nonlinear, non-convex manner, an arbitrary implementation of a deterministic optimization approach can lead to local minima without expanding the LP model's potential to its full capacity. As a result, the algorithm used for the solution of the optimization problem of Eq. (17) consists of the combined efforts of a deterministic search method operating in a local level and a general plan operating in a global level. The local search method most efficiently suited for Eq. (17) is the interior point trust region approach proposed by Coleman, T. F. and Li, Y. [14]. According to the approach, the quadratic trust region sub-problem is approximately solved as the minimization of a quadratic problem subjected to an appropriate ellipsoidal constraint. The global level general plan is achieved through the multiple execution of the interior point trust region method, initiated from different stochastically generated locations inside a prediction region (Figure 2).



Figure 2. Multistart algorithm used for the calibration process

The termination criteria for the proposed optimization scheme consists of an iterative evaluation of the objective function value for the normalized static stiffness of the LP model, along with a maximum boundary on the number of the overall sampling points x_i .

5. SINGLE INTERFACE DOF NUMERICAL VERIFICATION

As presented in the previous paragraphs, the proposed procedure can provide a computationally viable alternative to the direct FEM approach for the solution of the SSI phenomenon. However, the procedure is founded on a number of assumptions, which can render its efficiency questionable. To this end, it is essential to verify the proposed procedure through a realistic SSI example. The verification of the proposed procedure is accomplished through the case study of a strip foundation on homogenous clay soil as illustrated in figure 3. In addition to the proposed inelastic LP model approach (M.1), a simplified case of the presented LP assembly with conventional elastic complementary components tuned only to the elastic impedance function of the condensed segment (M.2) is also implemented for reasons of efficiency comparison.



Figure 3.: Plane strain shallow foundation example

As a targeted behavior is essential to the case study, a Finite element model of the soil foundation segment has been constructed. The soil foundation system is simulated as a plane strain model with a rigid element strip foundation. An elastoplastic constitutive law is implemented for the representation of the soil according to an associative plasticity flow with isotropic hardening and Von-Mises as the law's yielding criterion. The Soil constitutive law properties are summarized in table I. The overall FEM model includes a 100m x 50m truncated region of the semi-infinite soil domain, where absorbing boundaries are introduced at the side of the model according to [15], while a rigid bedrock is simulated at the 50m depth of the model.

Table I . Properties of the selected soil materials

Yielding Criterion	E(Mpa)	Hardening Param. Ho	Poisson ratio v	Yield Stress in Pure shear fy (Mpa)
Von Mises	200000	1000	0.3	200

For the current study only one DOF is considered as an interface between the soil foundation segment and the superstructure, the horizontal displacement of the centroid of the

strip foundation. As a result a single DOF elastoplastic constitutive law is utilized as the base component of the LP model of both the proposed method (M.1) and the conventional complementary component method (M.2). The selected constitutive law is described by the following elastoplasticity loading/unloading conditions (eq.18) and yielding equation (eq.19).

$$\dot{\gamma} \ge 0$$
, $\Phi(F) \le 0$, $\dot{\gamma} \cdot \Phi(F) = 0$ (18)

$$\Phi(F) = \left| F - H(e_p) \right| - G(a) \tag{19}$$

F denotes the generalized force of the base component while Φ is the yielding criterion of the elastoplastic constitutive law. In regard to the terms of the yielding function, H corresponds to the kinematic hardening function, G corresponds to the isotropic hardening function and e_p and a are isotropic and kinematic hardening parameters. A polynomial form is selected for the representation of the hardening function G and H, where the polynomial factors are calibrated according the quasi-static behavior of the FEM model.Results of the base component efficiency in regard to quasi static loading are illustrated in figure 4.



Figure 4.: Efficiency of the simplified and proposed LP models under quasi-static excitation for a) displacement to pseudotime b)and Force to displacement relation

For this single interface DOF the system's dynamic traits are extracted from the FEM model for different values of the interface DOF displacements in the range of 0 to 60mm. The condensed segment's impedance function and returning force matrix doublets $S_{reduced}$ and RF are extracted through the appropriate linearization and dynamic condensation of the system in the frequency domain in regard to the preselected interface DOF. The extracted properties are used as a target for the calibration of the complementary components of the proposed method (M.1), while the simplified method (M.2) is tuned according to the elastic impedance function of the targeted system.

5.3 Time domain verification of the proposed procedure

As both LP methods (M.1) and (M.2) fulfil the acceptable accuracy criteria for the quasi static behaviour of the targeted soil-foundation system, a dynamic case study analysis is performed in the following paragraphs. The dynamic response of the soil-foundation interface DOF under harmonic excitations on a frequency range of 0.5-4Hz is compared between the two LP model reduction methods and the actual FEM model as illustrated in (fig. 6-9).



Figure 7. Low intensity dynamic response of LP models, Force to displacement relationship



Figure 8. High intensity dynamic response of LP models, displacement to time relationship



Figure 9. High intensity dynamic response of LP models, Force to displacement relationship

As observed in figures (fig.6-7) both the proposed method and the simplified approach emulate the elastic dynamic behavior of the targeted FEM model in an accurate manner, as elastic dynamic properties are targeted by both models during the calibration process. On the scenario of higher excitation intensity, the subsoil region demonstrates permanent plastic deformation as illustrated in figures (fig. 8-9). It is observed that within the selected frequency range 0.5-4Hz, the post yield behavior of the simplified LP approach (M.2) is less efficient in comparison to the proposed method. This can be attributed to the fact that (M.2) method's limited tuning process of the conventional complementary components cannot maintain a behavioural accuracy in intensity or frequency regions excluded from the calibration process. Additionally as expected the observed error is higher for higher frequencies where the static components of the reduced methods is less dominant.

5. CONCLUSIONS

A lumped parameter modelling method has been proposed in the current paper, capable of accurately emulating the dynamic behaviour of the soil foundation system. In contrast to methods presented in the past literature, the proposed method copes with the frequency depended properties of the soil-foundation system in different intensity regions through the expansion of the lumped parameter model framework to inelastic dynamic systems. The procedure is numerically verified in both quasi static and dynamic excitations through the comparison of the FE model, the complete and the simplified version of the proposed approach. The results are limited to single DOF representations of simplified soil foundation systems and thus future work is essential to the expansion of the proposed method to realistic soil foundation systems.

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